



L^AT_EX Note Template

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Github: <https://github.com/fenglielie/latexzero>

1 Theorem, Proposition, Proof

Theorem 1.1. *If $1 < p < \infty$ and $m > n/p$, or $p = 1$ and $m \geq n$, there exist a constant $C = C(m, n, \gamma, p)$, such that*

$$\|R^m u\|_{L^\infty(\Omega)} \leq C d^{m-n/p} |u|_{W_p^m(\Omega)}$$

for all $u \in W_p^m(\Omega)$.

Proof. First, we assume that $u \in C^m(\Omega) \cap W_p^m(\Omega)$. We can use the pointwise representation of $R^m u(x)$.

$$\begin{aligned} |R^m u(x)| &= m \left| \sum_{|\alpha|=m} \int_{C_x} k_\alpha(x, z) D^\alpha u(z) dz \right| \\ &\leq C \sum_{|\alpha|=m} \int_{\Omega} |x - z|^{-n+m} |D^\alpha u(z)| dz \\ &\leq C' d^{m-n/p} |u|_{W_p^m(\Omega)}. \end{aligned}$$

The proof can be completed via a density argument. □

Theorem 1.2 (xxx). *If $1 < p < \infty$ and $m > n/p$, or $p = 1$ and $m \geq n$, there exist a constant $C = C(m, n, \gamma, p)$, such that*

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Proposition 1.3.

$$Q^m u(x) = \sum_{|\lambda| < m} \left(\int_B \psi_\lambda(y) u(y) dy \right) x^\lambda$$

where $\psi_\lambda \in C_0^\infty(\mathbb{R}^n)$ and $\text{supp}(\phi_\lambda) \in \bar{B}$.

Proof. This follows from xxx if we define

$$\psi_\lambda(y) = \sum_{\alpha \geq \lambda, |\alpha| < m} \frac{(-1)^{|\alpha|}}{\alpha!} a_{[\lambda, \alpha - \lambda]} D^\alpha (y^{\alpha - \lambda} \phi(y)).$$

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Proposition 1.4 (xxx).

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2 Corollary, Lemma, Claim

Corollary 2.1. *Under the assumption of xxx, the following inequality holds*

$$\inf_{v \in P^{m-1}} \|u - v\|_{W_p^k(\Omega)} \leq C_{m,n,\gamma} d^{m-k} |u|_{W_p^k(\Omega)}, \quad k = 0, 1, \dots, m,$$

Corollary 2.2 (xxx). *Under the assumption of xxx, the following inequality holds*

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Lemma 2.3. Let $f \in L^p(\Omega)$ for $p \geq 1$ and $m \geq 1$ and let

$$g(x) = \int_{\Omega} |x - z|^{-n+m} |f(z)| dz$$

Then

$$\|g\|_{L^p(\Omega)} \leq C_{m,n} d^m \|f\|_{L^p(\Omega)}.$$

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Claim 2.5. $Q^m u$ is a polynomial of degree less than m in x .

Claim 2.6 (xxx). $Q^m u$ is a polynomial of degree less than m in x .

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3 Definition

Definition 3.1. Ω is star-shaped with respect to the ball B if , for all $x \in \Omega$, the closed convex hull of $\{x\} \cup B$ is a subset of Ω .

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4 Example

Example 4.1. The integral form of the Taylor remainder for $f \in C^m([0, 1])$ is given by

$$f(s) = \sum_{k=0}^{m-1} \frac{1}{k!} f^{(k)}(0) + \int_0^s \frac{1}{(m-1)!} f^{(m)}(t)(s-t)^{m-1} dt.$$

Example 4.2 (xxx). The integral form of the Taylor remainder for $f \in C^m([0, 1])$ is given by

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5 Problem, Solution

Problem 5.1. Calculate the integral of the function $g(x) = 3x^2$ with respect to x .

Solution 5.1. To calculate the integral of $g(x) = 3x^2$, we use the power rule for integration:

$$\int 3x^2 dx = x^3 + C$$

where C is the constant of integration. □

Problem 5.2 (xxx). Calculate the integral of the function $g(x) = 3x^2$ with respect to x .

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6 Remark

Remark 6.1. Such a polynomial is not unique, due to the choice of cut-off function ϕ .

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7 Note

Note 7.1. The degree of $Q^m u$ is at most $m - 1$.

Note 7.2 (xxx). The degree of $Q^m u$ is at most $m - 1$.

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8 lstlisting

Listing 1: hello world

```
1 def hello():
2     print("Hello, world!")
3
4 hello()
```

Listing 2: hanoi.py

```
1 step = 1
2
3
4 def hanoi(n, a, b, c, depth=0):
5     def move(n, a, c):
6         global step
7         print("      " * depth, end="")
8         print(f"{step=:} move [{n}] from {a} to {c}")
9         step += 1
10
11     if n == 1:
12         move(n, a, c)
13     else:
14         hanoi(n - 1, a, c, b, depth=depth + 1)
15         move(n, a, c)
16         hanoi(n - 1, b, a, c, depth=depth + 1)
17
18
19 if __name__ == "__main__":
20     n = int(input("Hanoi Problem, N = "))
21     hanoi(n, "A", "B", "C")
```

9 algorithm

Algorithm 1: what

Input: This is some input

Output: This is some output

/ This is a comment */*

```
1 some code here;
2  $x \leftarrow 0$ ;
3  $y \leftarrow 0$ ;
4 if  $x > 5$  then
5   |  $x$  is greater than 5 ;
6 else
7   |  $x$  is less than or equal to 5;
8 end
9 foreach  $y$  in 0..5 do
10  |  $y \leftarrow y + 1$ ;
11 end
12 for  $y$  in 0..5 do
13  |  $y \leftarrow y - 1$ ;
14 end
15 while  $x > 5$  do
16  |  $x \leftarrow x - 1$ ;
17 end
18 return Return something here;
```
